Introduction: The Significance of Paul A. Samuelson in the Twenty-First Century

The contributors to this volume had unambiguous foundations upon which to build, due to Samuelson's use of mathematics as a language, "physics as the science for economics to imitate" (Hayek, 1992, p. 5), and reality over theory as a paradigm. Samuelson's early and later writings are consistent and somewhat invariant, and in harmony with Adam Smith's maximizing individual in society. Samuelson asked: "What is it that the scientist finds useful in being able to relate a positive description of behavior to the solution of a maximizing problem? That is what a good deal of my own early work was about. From the time of my first papers on 'Revealed preference' . . . through the completion of *Foundations of Economic Analysis*, I found this a fascinating subject . . . my positive descriptive relations could be interpreted as the necessary and sufficient conditions of a well-defined maximum problem" (Samuelson, 1972, p. 3).

Samuelson developed broad frameworks such as the neoclassical synthesis, a mixed economy, and the surrogate production function, which provided practitioners with a vision for research. His contributions to economics are rich, complex, heavy with facts, consequential, and relevant to the ordinary economics of life. Because of the quality of Samuelson's output and methods, the contributors to this volume see a near complete success for his theories in the twenty-first century.

Many of the contributors have defended Samuelson's work elsewhere. Now they have gathered to appraise the relevance of his work in the twenty-first century. Robert Solow explicitly states that Samuelson's Overlapping Generations Model (OLG) had slipped through the cracks in a previous work, which he now seeks to remedy in this volume. Luigi Pasinetti demonstrates elements of similarity between Samuelson's and Piero Sraffa's writings to explain why they were friends and not enemies, as would be expected. Geoff Harcourt discusses Samuelson's repeated and

vexed interest in Karl Marx's approaches, and identifies an "Aha!" moment about how Samuelson treats the transformation problem.

Appraising Samuelson: Units of Appraisal

As "Archimedes' lever is useless without a fulcrum to rest it on, and . . . angels need the point of a needle to dance upon" (Samuelson, 1978, p. 790), so too do we need a template against which to appraise the significance of Samuelson's writings for twenty-first century economists. Samuelson provided some insight when he argued that in order to appraise Alfred Marshall's originality we must take into consideration economists such as John Stuart Mill and Antoine-Augustin Cournot, whose contributions Marshall knew well (Samuelson, 1972, p. 22). If we were to sample the writings that Samuelson knows well, then mathematicians such as Henri Poincare, Frank Ramsey, and John von Neumann, scientists such as Albert Einstein, James Clerk Maxwell, and Henri-Louis Le Chatelier, philosophers such as Ernst Mach, Karl Popper, and Thomas Kuhn, and economists such as Adam Smith, David Ricardo, Karl Marx, and John Maynard Keynes would be included.

In awarding the Nobel Prize to Samuelson in 1970, the Nobel committee identified works that were worthy of appraisal. They were "*scientific work through which he has developed static and dynamic economic theory and actively contributed to raising the level of analysis in economic science*." These include Samuelson's novel view that "under free trade both parties are better off than under no trade at all, but are not necessarily in the optimum position" (Samuelson, 1966, p. 779); his Le Chatelier principle that explains how an economic system that is in equilibrium will react to a perturbation; his Samuelson–Bergson utility function that measures welfare gains; and his factor price equalization theory, which as John Hicks pointed out, "if there is a removal, not only of the obstacles to free trade, but also of the obstacles of factor movement . . . the two economies then become virtually one economy" (Hicks, 1983, p. 235). Those enumerated novelties are in addition to his neoclassical synthesis, revealed preference, multiplier-accelerator, and surrogate production function models. These novel contributions have already withstood the rigorous tests of coherency, consistency, falsification, and pragmatism, and have become objective and valid scientific achievements because they are open for revision and criticism. For over half a century, Samuelson's contributions have held up well with much cross-fertilization from other areas. In sum, as Kenneth Arrow puts it, "Samuelson is one of the greatest economic theorists of all time" (Arrow, 1967, p. 735). He should, therefore, be appraised as such.

An appraisal is different from a mere description. It appraises why a theory is superior and does not bother about how to construct an even more superior theory (Latsis, 1980, p. 3). Samuelson's hypotheses fit in this appraisal schema because they are scientific. Karl Popper draws a separating line between the "empirical sciences on the one hand, and mathematics and logic as well as 'metaphysical systems' on the other," and called this separation "the *problem of demarcation*" (Popper, 1968, p. 35). The fact that mathematics is included in the nonscience side should not be perceived as having a negative implication for economics and Samuelson's work. T. W. Hutchison cites Popper as saying that "The success of mathematical economics shows that one social science at least has gone through its Newtonian revolution." He adds that "surely such an outstanding post-Newtonian salient would deserve the closest analysis and appraisal from philosophers of science" (Hutchison, 1980, p. 187). Popper fuses the idea of "corroboration" with the idea of "appraisal" when he notes that "corroboration can only be expressed as an appraisal" (Popper, 1968, p. 265). Mark Blaug states that "By the degree of corroboration of a theory, I mean a concise report evaluating the state (at a certain time *t*) of the critical discussion of a theory, with respect to the way it solves its problems; its degree of testability; the severity of the test it has undergone; and the way it stood up to these tests. Corroboration (or degree of corroboration) is thus an evaluating *report of past performance*" (Blaug, 1983, p. 26).

A "corroborative appraisal" establishes a fundamental relationship between accepted basic statements and the hypothesis (Popper, 1968, p. 266). Hypotheses "are 'provisional conjectures' (or something of the sort); and this view too, can only be expressed by way of an appraisal of these hypotheses" (Popper, 1968, p. 265). "From a new idea . . . conclusions are drawn by means of logical deduction. These conclusions are then compared with one another and with other relevant statements, so as to find what logical relations (such as equivalence, derivability, compatibility, or incompatibility) exist between them" (Popper, 1968, p. 32). Therefore, Popper's demarcation criteria are the "standards for appraising competing scientific hypotheses in terms of their degrees of verisimilitude" (Blaug, 1983, p. 10). Hypotheses must be subjected to a severe test and found compatible, that is, not falsified. The testing procedure is fourfold: (1) A test of the internal consistency of the system by comparing conclusions among themselves, (2) A test of the logical form of the theory to see if it is empirical or tautological, (3) A comparison of the theory with others to see whether it truly is a "scientific advance should it survive our various tests," and (4) A test of the theory "by way of empirical applications of the conclusions which can be derived from it" (Blaug, 1983, pp. 32–34). The overall implication is that if Samuelson's theories are incompatible, we can regard

them as falsified. If they are compatible, then we might give them some degree of "positive corroboration," but that will depend on the "severity of the various tests" that the hypothesis has passed (Blaug, 1983, pp. 266–267). A positive degree of corroboration is like a nested function: Positive corroboration = f [severity of test = g (degree of testability = h {simplicity of the hypothesis})].

Although Samuelson has some grand unifying scientific theories such as the neoclassical synthesis, he has also made particular scientific contributions to many subdisciplines in economics that need to be appraised as well. Fortunately, appraisal methodology comes in "units" as well as in bundles of "units." Imre Lakatos, a student of Popper, called units of appraisal in science a "research program" or a series of connected theories, rather than a single theory (Lakatos and Musgrave, 1977). A single theory can be falsified when only one instance of refutation appears. However, a research program is not easily falsified. It has theories in its "hard core" that practitioners are not willing to abandon, and theories in its "protective belt" that practitioners are interested in improving. The Duhem–Quine hypothesis also embraces units of appraisal. It holds that the incompatibility of one consequence does not falsify all of the antecedents (Quine, 1990, pp. 13–14). For example, if one finds that the paradox of thrift hypothesis is incompatible, one does not have to give up the neoclassical synthesis. For Samuelson, such units of appraisal are exemplified in the roles that mathematics and facts play in economics. Building on Pareto's idea that mathematics represents complexly interacting and independent phenomena, Samuelson adds that "after mathematical notions have performed the function of reminding us that everything depends upon everything else, they may not add very much more—unless some special hypothesis can be made about the facts" (Samuelson, 1966, p. 1758).

Besides the above broad methodological perspectives, Samuelson can be appraised as a "craftsman" using his personal knowledge to improve economic science. A "personal appraisal" holds that "in every act of knowing there enters a passionate contribution of the person knowing what is being known" (Polanyi, 1958, p. viii). As Jerome R. Ravetz (1979, p. 75) frames the problem, we are interested in how "objective scientific knowledge can result from the intensely personal and fallible endeavor of creative scientific inquiry." On the craftsman's side of appraisal, Samuelson has certainly demonstrated high "morale" defined as "any positive and energetic attitude toward a goal" (Bateson and Mead, 1941, p. 206). The contributors of this book are living proof of successful "morale transfer," and good

"morale resonance" among colleagues and students with Samuelson's craftsmanship. But besides craftsmanship and good morale, Samuelson can be appraised from the perspective of "universally valid appraisals" (Polanyi, 1958, p. 22), "systems of appraisals" (Polanyi, 1958, p. 43), and "appraisal of order" (Polanyi, 1958, p. 36), the criteria of which include "(1) a correct satisfaction of normal standards, (2) a mistaken satisfaction of normal standards, and (3) action or perception satisfying subjective, illusory standards" (Polanyi, 1958, p. 363). The neoclassical syntheses and intergenerational models are objective and valid scientific achievements because they are open for revision and criticism from everyone, and have had significant and verified progress thus far.

Samuelson should further be appraised for his adoption of successful epistemological viewpoints in his scientific approach to economics. The big "M" approach of Rod Cross places Samuelson's works beside those of Mach. Samuelson's method is also consistent with the epistemological approach of Poincare and Einstein. Poincare would build up science from lower level hypotheses, such as Galileo Galilei's one-dimensional to two-dimensional motions of falling bodies, ending up with Isaac Newton's laws. Einstein would make inference statements from axioms, but in moving from axiom to inference statements one has a clear link with the empirical world of data and experiments, including mental (*Gedanken*) experiments (Miller, 1984, pp. 39–46). Such research programs are geared to find novel facts. "Einstein's program . . . made the stunning prediction that if one measures the distance between two stars in the night and if one measures the distance between them during the day (when they are visible during an eclipse of the sun), the two measurements will be different" (Lakatos, 1980, p. 5). As Popper indicates, "stars close to the sun would look as if they had moved a little away from the sun, and from one another" (Popper, 1963, p. 37). This is indeed a novel prediction of Einstein's program. For Poincare, "facts outstrip us, and we can never overtake them; while the scientist is discovering one fact, millions and millions are produced in every cubic inch of his body" (Poincare, 1908, p. 16). We must therefore select facts. "The most interesting facts are those which can be used several times, those which have a chance of recurring" (Poincare, 1908, p. 17). Recurring facts are "simple" such as the stars, the atoms, and the cell. In short, simple facts lie "in the two extremes, in the infinitely great and in the infinitely small" (Poincare, 1908, 18–19). One is reminded of Samuelson's Simple Mathematics of Income Determination, Surrogate Production Function, and simple 2×2 trade models. To sum up, in Samuelson's own words, "we theorists like to work with extreme polar

cases, what is the natural model to formulate so as to give strongest emphasis to external effect?" (Samuelson, 1966, p. 1235).

On the empirical side, Samuelson is primarily appraised for his study of the mixed economy. "The mixed economy is not a very definite concept. I have purposely left it vague, in part because that is its intrinsic nature and in part because increased precision should come at the end rather than at the beginning of extensive research" (Samuelson, 1966, p. 632). Further, "A mixed economy in a society where people are by custom tolerant of differences in opinion, may provide greater personal freedom and security of expression than does a purer price economy where people are less tolerant" (Samuelson, 1972, p. 628). Even though "History oscillates, backtracks, and spirals" (Samuelson, 1972, p. 612), in the end the mixed economy emerges because it is based on reality. We know that Samuelson nourishes his foresight with strong doses of the reality paradigm, for he said that "I would take aid from the Devil if that would help crack the puzzle of economic reality" (Samuelson, 1986, p. 873), and that "it is better to have a model with inexact foundations that gives you a good grip to handle reality than to wait for better foundations or to continue to use a model with good foundations that is not usefully relevant to explain the phenomena that we have to explain" (Samuelson, 1986, p. 295). In short, "A good economist has good judgment about economic reality" (Samuelson, 1972, p. 775).

On the theoretical side, Samuelson is also most appraised for how his theories explain reality. For his theories to survive in the twenty-first century, they must solve and explain problems and anomalies in a normal scientific way. Normal science will require us to: (1) unearth economic values; (2) compare theories T_n with T_{n-1} to see which one performs better; and (3) overthrow or re-specify degenerating theories.[1]

Samuelson's neoclassical synthesis can be used to appraise different theories to see which is better. Take the demand versus supply-side theories of the 1970s and 1980s for example. The demand-side theory was well tested in the 1970s against the stagflation problem, and some economists charged that it was weakening. Other Keynesian ideas such as the paradox of thrift were eliminated from some of the newer editions of Samuelson's own book, *Economics*. Some major principle textbooks dropped the Keynesian IS/LM and the Keynesian-cross diagrams. However, the hard-core elements of the demand-side system such as the mixed economy, wage rigidity, and interest inelasticity remain intact, and its future seems assured: "the underlying framework for a new period of creative consensus in economic thought . . . will be a newly appraised balance between the

public and private sectors in which the role of the former is considerably elevated over its earlier status" (Heilbroner and Milberg, 1995, p. 119). Newer policy problems such as targeted interest rate policies and monetary policies within the EMU saw the need to keep the Keynesian modified paradigm within the Mundell–Fleming BOP/IS/LM model, and its implied Keynesian-cross framework. This example underscores the fact that the Keynesian paradigm has a strong heuristic power to suggest ways of solving a wide variety of problems; that it is theoretically progressive in that it will yield more testable content in T_n than in T_{n-1}; and that it is empirically progressive, for at least some of its additional testable consequences can be confirmed.

As an appraisal of Samuelson's ability to re-specify a weakening theory, let us consider his improvement of the Heckscher–Ohlin theory. As a criterion for a better theory, one may be content to accept von Mises' "Science and Value" concept, "a situation in which a given amount of capital and labor was able to produce a definite quantity of material economic goods 'better' than a situation in which the same amount could produce only a smaller quantity" (von Mises, 1960, p. 36). But Samuelson would not settle there: "What Samuelson did was graft Ohlin's trade theory and the problems connected with its rigorous articulation and generalization onto the mainline research tradition concerned with the conditions governing the existence, uniqueness and stability of general competitive equilibrium" (De Marchi, 1976, pp. 112–113).

To sum up, we have been very restrictive in our appraisal of Samuelson's work for the twenty-first century, limiting our examples to only the mixed economy and policy combinations. The contributors to this volume are more impressive in their predictions and explanations of their topics, which include ideas, theories, and facts.

Samuelson Appraised Through His Own Methodology

In at least two articles, Samuelson uses the word "appraisal" explicitly in the title. In one, "Economics of Futures Contracts on Basic Macroeconomic Indexes: An Economist's Appraisal" (Samuelson, 1986, p. 557), he reaches the conclusion that "Economists' theories will be better when they can perceive what the *expected rate of inflation* really is . . . . What is infinitely more important, the players in the economic game . . . will be better off when the exchange institutions evolve that help to signal risks, reduce them, and allocate the residual irreducible risks optimally"

(Samuelson, 1986, p. 558). In the other, "2 Nobel Laureates' Theories on Trade: An Appraisal" (Samuelson, 1986, p. 831), where, after his appraisal of the contributions of J. E. Meade and Bert Ohlin, he reaches the conclusion that "Each man has demonstrated that those who are best at pure science are often outstanding policy advisers and public servants" (Samuelson, 1986, p. 831).

One can surmise that Samuelson uses the term "appraisal" broadly in accordance with the methodologies described above. It could include his way of appraising two competing paradigms, such as when he writes: "The two paradigms seem to tell different stories. Which is the relevant story, the correct one for the case of competitive capitalism? There can be no doubt as to the answer. The bourgeois paradigm of Smith, Ricardo, Piero Sraffa, and Leon Walras correctly predicts that the new invention will displace the old technique under ruthless competition" (Samuelson, 1986, p. 365). It could also mean finding necessary and sufficient conditions, and logics to appraise a theory: "Here I provide necessary and sufficient conditions for price invariance in the presence of exponentially depreciating durable capital goods. The result is a surprisingly simple criterion: price ratios are invariant to interest rate changes if and only if all industries have . . . the same capital-to-labor ratio" (Samuelson, 1986, p. 375).[2]

Fritz Machlup distinguishes three streams of Samuelson's methodology: (1) theories, antecedents, and consequents that must be mutually implicative, and identical in meaning; (2) strong simple cases like the 2×2 trade model that bring out elements of truth in a complex theory; and (3) methods as advocated in his *Foundations of Economic Analysis* that emphasize the derivation of "operationally meaningful" theorems (Samuelson, 1972, p. 758). We look briefly at these three streams below.

Mutually Implicative Theories

Regarding mutually implicative theories, Samuelson wrote that "after mathematical notions have performed the function of reminding us that everything depends upon everything else, they may not add very much more—unless some special hypothesis can be made about the facts" (Samuelson, 1966, p. 1758). Again, "mathematics is neither a necessary nor a sufficient condition for a fruitful career in economic theory" (Samuelson, 1966, p. 1760). Furthermore, "Marshall in his own way also rather pooh-poohed the use of mathematics. But he regarded it as a way of arriving at the truths, but not as a good way of communicating such truths" (Samuelson, 1966, p. 1755).

2 × 2 Trade Theory

Nowhere is the process of scientific appraisal more clearly demonstrated than in the trade theory of economics. Trade theory, from mercantilism to Adam Smith's absolute advantage, and from Ricardo's comparative advantage to Heckscher–Ohlin's factor proportion–intensity assumptions, have been now standardized in a 2 × 2 form (two goods and two factors). Samuelson's place in this research is secured when he demonstrates that "Both the classical and Ohlin versions of the explanation of trade may thus be viewed as adaptations of a common general equilibrium framework" (De Marchi, 1976, p. 112). Samuelson has shown that "the assumptions sufficient to yield factor price equalization also suffice . . . to yield the Heckscher-Ohlin theorem" (De Marchi, 1976, p. 112). In scientific parlance, Samuelson has shifted the research in trade theory from "before Samuelson" to "after Samuelson." This shift has made it possible for "rigorous articulation and generalization on the mainline research tradition concerned with the conditions governing the existence, uniqueness, and stability of general competitive equilibrium" (De Marchi, 1976, p. 113). Therefore, under the idea of appraisal that asks "Is theory A better than theory B?" (Latsis, 1976, p. 3) it is fair to say that Samuelson's research on the 2 × 2 trade theory is poised for scientific conjectures and refutations in the twenty-first century.

Operationalism

A byproduct of Einstein's Special Theory of Relativity is that it allows us to define a term by the physical operations the agent performs in order to observe the object. Any concept corresponds with a set of operations. Samuelson seeks "the derivation of operationally meaningful theorems" in economics (Samuelson, 1947, p. 3). One aim of meaningful theorems is that we can ascertain their truth values, the likelihood that they will occur in reality, and that we can attempt to verify them (Machlup, 1978, pp. 165–166). Another view is that through these operational theorems, we may not "verify" but "infer" the object.

According to Machlup's argument, operationalism purges a theory of its assumptions, and therefore of its theoretical and mathematical framework. Friedman's F-twist theory emphasizes the independence of assumptions from prediction. Others, such as Bateson and Mead (1941, p. 55), think that "whenever we start insisting too hard upon 'operationalism' or symbolic logic or any other of these very essential systems of tramlines, we

lose something of the ability to think new thoughts. And equally, of course, whenever we rebel against the sterile rigidity of formal thought and exposition and let our ideas run wild, we likewise lose. As I see it, the advances in scientific thought come from a *combination of loose and strict thinking*, and this combination is the most precious tool of science."

To cut through the hurdles of the various brandings of his methodology, Samuelson reminds us that he is a truth seeker. On the one hand he argues for reality as the paradigm, but on the other hand he holds that "observations are not merely seen or sensed but rather often are perceived in gestalt patterns that impose themselves on the data and even distort those data" (Samuelson, 1993, p. 244). Here we have the interplay between theory and fact. We know that his basic paradigm of facts is not based on dialectic but on the cumulative method.[3] As Samuelson puts it, "in the language of [Thomas] Kuhn, knowledge in economics accumulates, and paradigms can be commensurable only if the 'black' in one paradigm can be considered as the 'white' in another paradigm" (Samuelson, 1993, p. 244).

Analysis of Samuelson's Specific Contributions

In this section, we look at some of Samuelson's specific contributions to highly specialized topics in economics. The idea is to underscore the essence of the contributions in relation to the respective authors' appraisal as to why the material will be relevant for the twenty-first century.

Overlapping Generation Models

The first section of this book contains three chapters:

- "Overlapping Generations," Robert M. Solow
- "Paul Samuelson's Amazing Intergenerational Transfer," Laurence J. Kotlikoff
- "Social Security, the Government Budget, and National Savings," Peter Diamond

Samuelson introduced the overlapping generations model (OLG) to "develop the equilibrium conditions for a rational consumer's lifetime consumption-saving pattern" (Samuelson, 1958, p. 104). Since that time, the OLG has become a strong rival theory to the Arrow–Debreu general equilibrium model, extending theoretical and empirical research to areas that previously could not be reached.

Solow's description of the original Samuelson model invites the distinction between theoretical and practical uses of the model. Essentially, OLG has an overlapping generation structure, which operates in infinite time. Although no generation can step into the same time twice, a person in one generation, G^t, can trade with a person in another generation, G^{t-1}, G^{t-2}, etc., where a "household born at t will be said to be of generation t, or simply G^t" (Hahn and Solow, 1995, p. 13). For example, a person in a three-period model that wants to save for retirement in the next period can lend to a person in the first period. This is essential because the model deals with generations which are defined in terms of age, and people have different tastes and preferences over different age cohorts.

A similar model is found in the appendix of Maurice Allais' book, *Économie et Intérêt* (1947). Both Solow and Malinvaud (1987) essentially agree by implication. They agree that time is infinite in both cases, but Allais uses two periods and Samuelson uses three periods. In Allais' model, the presence of Government is as a wealth holder and/or a debtor, say of land and financial assets, following a variety of hypotheses (Malinvaud, 1987, p. 104). According to Kotlikoff, in Samuelson's model, "government can redistribute across generations." The interest rate in Allais' model "is not fully determined by the individual's preference for the present and by technical feasibilities" (Malinvaud, 1987, p. 104), and, according to Samuelson, it can have a biological dimension that results when a correlation between prices and population changes leads to equality between the interest rate and population growth rate.

The interest rate as a bridge between current and postponed consumption appears as a time cohort model. To turn this into a real OLG model, we have to layer it with people in different age cohorts. Using the didactics of Samuelson's three-period model, the middle generation may want to lend excess current income (savings) to the younger generation, but not to the old, for when the middle generation becomes old, it wants to consume its savings plus earned interest. Therefore, consumption, c, and savings, s, cannot exceed wealth, w, that is, $c + s \leq w$. Solow's chapter continues in this line of symbolic generalization, originating in his joint paper with Frank Hahn, to illustrate market imperfections (Hahn and Solow, 1995, p. 13).

Kotlikoff's work has given rise to a new term in the expansion and articulation of the OLG model, particularly in generational accounting. Both Kotlikoff and Diamond take up current and future concerns of the Social Security problem, a good indication of the relevance of the model for the twenty-first century.

Expectation, Uncertainty and Public Goods

The second section of this book contains two articles:

- Prospective Shifts, Speculative Swings: "Macro" for the 21st Century in the Tradition Championed by Paul Samuelson, Edmund S. Phelps.
- "Paul Samuelson and Global Public Goods," William D. Nordhaus

Edmund Phelps appraises Samuelson from the point of view of expectation and uncertainty. Although his multiplier-accelerator contribution to Keynesian modeling became famous, his interest in macroeconomics ran deep and spanned the perspectives of theorists before and after Keynes. Ever interested in real-life economies, Samuelson was alert to the occasional sea change in the American economy. In the 1970s, he invented the term "stagflation" to stand for the ratcheting down to slower growth and increased unemployment amidst no lessening of inflation. Evidently parametric shifts were occurring, from the world price of energy to the exhaustion of the stock of unused ideas. Comparing this history with the stationary rational-expectations models becoming fashionable in the 1980s, Samuelson concluded that "[a]s a description of what happens in the real world and as a tool for intermediate-run macro predictions, the Lucas-Sargent-Barro model is a poor tool" (Samuelson 1986, Vol. V, 294).

Samuelson's concepts pertaining to a business economy – its orientation toward the future, the uncertainty of future prospect and the nonstationarity of its demographic and technological environment – were tools, Phelps argues, that Samuelson used and are tools we need in this century for an understanding of the secular shifts and big swings in the American economy. Samuelson does not see rational expectations as taking root in an enterprise economy. Samuelson, Phelps acknowledges, was "a pioneer of rational expectations theory" and willing to postulate rational expectations for the purpose of explaining differences in share prices across categories over a given period; and the "micro efficiency" he sees in those markets, if he is right, may justify the postulate for that purpose. Yet Samuelson finds "macro inefficiency" in the index of share prices. Thus rational expectations appear not to describe the ups and downs of the values entrepreneurs in aggregate put on new projects (and the values that retails investors place on the shares financing those projects).

Phelps, supporting Samuelson's views, points out that an enterprising economy is driven by the "visions" and "fears" of entrepreneurs, financiers and speculators. The postulate of rational expectations, Phelps argues, is simply inapplicable to the expectations of entrepreneurs peering into the

unknown. Furthermore, an outsider in the public has no way of forming rational expectations of the expectations of the entrepreneurs: Their knowledge is importantly "personal knowledge," which the public cannot access. And the public infer from its data available what the entrepreneurs must be expecting, for to suppose that their expectations could be the subject of unbiased estimates by the public would be to imply that entrepreneurs have no special knowledge and play no special role.

This piece recalls Keynes's point of view. In its essence, Samuelson's brand of expectation and uncertainty subscribes to Chapter 12 of Keynes's *The General Theory*. Keynes exposed a disequilibrium paradigm, in contrast to the rational expectation models that exposed an equilibrium paradigm.

Following Phelps' presentation, we need to understand the effects of the "visions and fears" excluded by the now-standard paradigm theory. Phelps examines the effects of expectations regarding three future events – future "debt bombs," future productivity surges, and war prospects. An economy continually pinged by such expectations is "never 'vibrating' up and down its saddle path"(Ibid.). Samuelsonian economics of the 21st century will emphasize that "macroeconomics must incorporate future prospects if it is to capture the big swings in economic activities." It will preserve the perspective that the rational-expectations movement excluded.

The distinction between public and private goods follows from concerns about market efficiency concepts. Nordhaus' piece fits in with the appraisal of polar cases of fact from an epistemological appraisal point of view. A public good, "for which the cost of extending the service to an additional person is zero," is "a polar case of an externality," says Nordhaus. Externality, nonrivalry, and nonexcludability are budding research programs, which will be of great concern for the twenty-first century because "private markets generally do not guarantee efficient outcomes." The "stock externalities" concern for public goods, particularly with regard to nuclear energy and greenhouse gases that have a firm hold in the twenty-first century policy concerns, and have implications for the course that international laws will take.

Revealed Preference and Consumer Behavior

There are two chapters in the third section of this book:

- "Revealed Preference," Hal R. Varian
- "Samuelson's 'Dr Jekyll and Mrs Jekyll' Problem: A Difficulty in the Concept of the Consumer," Robert A. Pollak

The explosion of research on revealed preference still builds on the foundations which Samuelson constructed. One foundation is the weak axiom of consumer behavior, another is the strong axiom, and a third is the fundamental theorem. While these axioms still support the model for empirical work, the empirical work now relates the strong axiom to Afriat's "cyclical consistency" criterion. Varian found that testing consistency with utility maximization requires a general axiom as well. The three axioms performed well, that is, "aggregate consumption data easily satisfied the revealed preference conditions." Besides consistency, Varian investigates the form, forecasting, and recoverability criteria. He concludes that the strong axiom is necessary and sufficient for utility maximization, and rich in empirical content.

Samuelson foresaw all the possible progressive birth signs of the revealed preference model when he initially proposed it. First, he gave it a twin feature (which he later integrated into a more single condition): (1) a single-value function on prices and income, subject to a budget constraint, and (2) homogeneity of order zero so as to make consumer behavior independent of the units of measurement of prices. Consider two batches or vectors of goods, ψ and ψ', with their respective price vectors, p and p', and denote their inner product by $[\psi p]$, and $[\psi' p']$, respectively. Now, we can observe the following: (3) "If this cost $[\psi' p]$ is less than or equal to the actual expenditure in the first period when the first batch of goods $[\psi p]$ was actually bought, then it means that the individual could have purchased the second batch of goods with the price and income of the first situation, but did not choose to do so. That is, the first batch (ψ) was selected instead of (υ')" (Samuelson, 1966, p. 7).

A third proposition deals with consistency. "If an individual selects batch one over batch two, he does not at the same time select two over one" (Samuelson, 1966, p. 7). In a later note, Samuelson compacts the first two propositions with the third; "Postulates 1 and 2 are already implied in postulate 3, and hence may be omitted" (Samuelson, 1966, p. 13).

At its inception, Samuelson had the foresight that the revealed preference model had some virtue: "even within the framework of the ordinary utility- and indifference-curve assumptions, it is believed to be possible to derive already known theorems quickly, and also to suggest new sets of conditions. Furthermore . . . the transitions from individual to market demand functions are considerably expedited" (Samuelson, 1966, p. 23). But the revealed preference theory matured into an even more powerful rival research paradigm. Samuelson wrote: "I suddenly realized that we could dispense with almost all notions of utility; starting from a few logical axioms of demand consistency; I could derive the whole of the valid utility

analysis as corollaries" (Samuelson, 1966, p. 90). The corollaries followed from axioms of consumer behavior noted above, which Varian has cast in more up-to-date symbolic form. But the way Samuelson stated them shows that he could be as eloquent in prose as in mathematics. The axioms as originally stated are as follows:

> Weak axiom: If at the price and income of situation A you could have bought the goods actually bought at a different point B and if you actually chose not to, then A is defined to be "revealed to be better than" B. The basic postulate is that B is never to reveal itself to be also "better than" A (Samuelson, 1966, p. 90).

> Strong axiom: If A reveals itself to be "better than" B, and if B reveals itself to be "better than" C, and if C reveals itself to be "better than" D, etc . . . , then I extend the definition of "revealed preference" and say that A can be defined to be "revealed to be better than" Z, the last in the chain. In such cases it is postulated that Z must never also be revealed to be better than A (Samuelson, 1966, pp. 90–91).

Samuelson then elevated the revealed preference theory to the empirical domain: "consumption theory does definitely have some refutable empirical implications" (Samuelson, 1966, p. 106), or we can "score the theory of revealed preference" (Samuelson, 1966, p. 106). Samuelson required a benchmark to allow refutation/scoring, for which he postulated this fundamental theorem: "Any good (simple or composite) that is known always to increase in demand when money income alone rises must definitely shrink in demand when its price alone rises" (Samuelson, 1966, p. 107). He then proceeded "to show that within the framework of the narrowest version of revealed preference the important fundamental theorem, stated above, can be directly demonstrated (a) in commonsense words, (b) in geometrical argument, (c) by general analytic proof" (Samuelson, 1966, p. 108).

How good a rival theory is the revealed preference theory? As Hildenbrand puts it, "Instead of deriving demand in a given wealth-price situation from the preferences, considered as the primitive concept, one can take the demand function (correspondence) directly as the primitive concept. If the demand function f reveals a certain 'consistency' of choices . . . one can show that there exists a preference relation . . . which will give rise to the demand function f" (Hildenbrand, 1974, p. 95).

Pollak's chapter describes a rich number of cases in which Samuelson distinguishes between the individual and the family as the consuming agent. On the theoretical side, he considers a Bergson–Samuelson type of social welfare utility function, with implications for Arrow's impossibility theorem. On the application side, he features Becker's "rotten kid" model problem, holding out the possibility of a solution with a family member as a possible dictator.[4]

Marxism

There is a single chapter in this section on Marx:

- "Paul Samuelson on Karl Marx: Were the Sacrificed Games of Tennis Worth It?" Geoff Harcourt

Is Samuelson a Marxist? The late Adolph Lowe said in one of his lectures that if Samuelson would say that Marx was right, then standard/orthodox economics would collapse. In Samuelson's words, "John Maynard Keynes was scientifically the greatest economist of this century. Only Adam Smith and Leon Walras can be mentioned in the same breath with him. Karl Marx can be mentioned in the same breath with Mohammed and Jesus, but it is of scientific scholarship that I speak and not of political movements and ideology" (Samuelson, 1986, p. 262). Again, Samuelson wrote: "I regard Marx as a scholar deserving of analysis on his objective merits and without regard to the deification or denigration meted out to him in various regions and ideologies." "I appraised Marx as a mathematical economist, . . . hailing Marx's most original contribution in *Capital*'s Volume II *Tableaus of Simple and Expanded Reproduction*. Marx's critics have missed this achievement, while at the same time his partisans have been praising his sterile paradigm" (Samuelson, 1986, p. 263). Also, "I follow Marx's portrait with Table II, which is what he and his followers wrongly think makes him great" (Samuelson, 1986, p. 273).

Harcourt challenges us to look at Samuelson's ideas from an eraser's point of view. When we look, we find Samuelson saying: "Contemplate two alternative and discordant systems. Write down one. Now transform by taking an eraser and rubbing it out. Then fill in the other one. *Voila*! You have completed your transformation algorithm" (Samuelson, 1972, p. 277). Here is a question and Samuelson's answer on this idea applied to Marx's transformation problem: "*The 'algorithmic transformation' from the 'value' model to the 'price' model (or vice versa), is truly a process of rejection of the former and replacement by the latter?* Here is my true crime. I pointed out the blunt truth. And this has been construed as an attack on Marx, covert or explicit" (Samuelson, 1978, p. 284). Naturally then, if Samuelson's idea is appraised as the truth, it shall prevail.

Stability

There is a single chapter in this section on Stability:

- "Paul Samuelson and the Stability of General Equilibrium," Franklin Fisher

Franklin Fisher wishes to attract attention to the topic of stability in the twenty-first century, for it has not yet reached a "satisfactory conclusion." Why is stability important? Not because it attracted the attention of Walras and Marshall (Walker, 1983, pp. 276–277). Stability is about "the determination of equilibrium values of given variables (unknowns) under postulated conditions (functional relationships) with various data (parameters) being specified" (Samuelson, 1966, p. 539). When rummaging into the equations of a theory, be it simple supply and demand equations, one always wants to know its stability properties, for if no more than equilibrium can be found, then "the economist would be truly vulnerable to the gibe that he is only a parrot taught to say 'supply and demand'" (Samuelson, 1966, p. 539). There would always be a need for stability analysis, even of a comparative static nature, to unearth the predictive power of models for scientific appraisals. Samuelson likened stability behavior to the soul and mind of business. "Since this competitive industry's comparative-statics can be shown to behave as if the industry had a soul and an integrated mind, expediency urges us to pretend it has" (Samuelson, 1986, p. 103). Those are, therefore, reasons to expect that this model will attract attention in the twenty-first century.

The sequel of statics versus dynamics also deserves unique treatment. Samuelson wrote: "For comparative-statics analysis to yield fruitful results we must first develop a theory of dynamics" (Samuelson, 1947, pp. 262–263). "Statics concerns with the simultaneous and instantaneous or timeless determination of economic variables by mutually interdependent relations . . . It is the essence of dynamics that economic variables at different points of time are functionally related . . . functional relationships between economic variables and their rates of change, their 'velocities,' 'accelerations,' or higher 'derivatives of derivatives'" (Samuelson, 1966, p. 354). There is a "formal dependence between comparative statics and dynamics . . . the *Correspondence Principle*" (Samuelson, 1966, p. 565). But there is a "two-way nature: not only can the investigation of the dynamic stability of a system yield fruitful theorems in static analysis, but also known properties of a (comparative) static system can be utilized to derive information concerning the dynamic properties of a system" (Samuelson, 1966, p. 565). "The nature of dynamic processes can best be appreciated from a study of concrete examples" (Samuelson, 1966, p. 593), and "in the field of pure theory, the important problem of the stability of equilibrium is wholly a question of dynamics. For it involves the question of how a system behaves after it has been disturbed into a disequilibrium state" (Samuelson, 1966, p. 613). How then do we involve facts and reality? George Feiwel (1982, p. 7) has a simple answer: "The growth of general equilibrium has given increased

focus to static concepts." No theory can be more static than Keynesian. "All sciences have the common task of describing and summarizing empirical reality. Economics is no exception" (Samuelson, 1966, p. 1756) and "no a priori empirical truths can exist in any field. If a theory has a priori irrefutable truth, it must be empty of empirical content" (Samuelson, 1966, p. 1757). Stability concerns are here to stay. All models of reality call upon them to assess their compatibility. Samuelson has grounded stability in dynamics, which is promising for the twenty-first century economics.

Keynes and Post-Keynesians

There are three chapters in the sixth section of this book:

- "Paul Samuelson and Piero Sraffa—Two Prodigious Minds at the Opposite Poles," Luigi Pasinetti
- "Paul Samuelson as a 'Keynesian' Economist," Lawrence R. Klein
- "Samuelson and the Keynesian/Post-Keynesian Revolution," Paul Davidson

Luigi Pasinetti appraises the source of Samuelson's interest in Piero Sraffa's work. He finds that the two authors were attracted to each other because their works are of "equally foundational character." Samuelson's *Foundations of Economic Analysis* (1947) is on the side of maximization under constraints, with the use of mathematics as a language, while Sraffa's *Production of Commodities by Means of Commodities* (1960) tries to weed out ambiguities in solving analytical problems, starting with a prelude and advancing to a more constructive stage. Although their approaches are different, "they reached the same analytical conclusion, though with different nuances, accentuation of details, or shades of emphasis" as it is illustrated by the debate on reswitching.

Where Samuelson and Sraffa differ is in methodology. Samuelson defends the "exchange paradigm," while Sraffa defends the "production paradigm." This disagreement is evident in Sraffa's work (1960), which Samuelson sees as a defense of Ricardo's labor theory of value. Pasinetti appraises Samuelson's labor theory view along with two other propositions, reaching a somewhat different conclusion in answer to the question, "Why has Samuelson been so interested in Sraffa?" Pasinetti argues that Samuelson's stated desire "of formulating a general theory of economic theories . . . , would seem to imply the absorption and inclusion also of Sraffa."

Klein's chapter "Paul Samuelson as a 'Keynesian' Economist" traces the development of Keynesian economics in the United States from the early days. Although Keynesian economics came into the United States in fragments, Samuelson views it as a unifying principle. "What made Keynes different . . . was the fact that . . . He tackled the whole thing in one brilliant analytic formulation and provided economists with a new way of looking at how the entire gross national product is determined and how wages and prices and the rate of unemployment are determined along with it" (Samuelson, 1986, p. 280).

One of Samuelson's first attempts in Keynesian economics is the simple mathematical formulation: "Y = C + I, and C = C(Y), I = Î" (Samuelson, 1966, pp. 1197–1219). Klein traces the development of "Paul Samuelson as a 'Keynesian' Economist," and also appraises the econometric foundation of Keynesian economics. Paul Davidson considers a different stream of post-Keynesian economics. Samuelson writes: "In . . . contrast to monetarism is the mainstream of modern economics today which . . . I shall call 'Post-Keynesian Economics.' Modern economics, as represented by men like James Tobin, Franco Modigliani, and myself, basically believes that changes in the money supply engineered by Federal Reserve policy have important effects upon the level of money, Gross National Product (GNP) and, depending upon the state of slackness in the employment market, upon real output and the price level" (Samuelson, 1978, p. 765). He then gives three post-Keynesian propositions "Even when the money supply is held constant" (1) Changes in thriftiness and the marginal propensity to consume can affect output, prices, and production; (2) An exogenous burst in I or investor's instinctive behavior has a systematic effect on GNP; and (3) An increase in public expenditure, or a cut in the tax rate has a systematic effect on GNP (Samuelson, 1978, p. 765). Davidson appraises the many versions of Keynes—Old Neoclassical, New Keynesians, Old Classical or New Classical Theorists—as adopting Keynesian general theory as their basic framework. As long as those research programs last, Samuelson's input into Keynesian economics will be followed.

International Economics and Finance

There are three chapters in this section of the book:

- "Paul Samuelson and International Trade Theory Over Eight Decades," Avinash Dixit

- "Paul Samuelson's Contributions to International Economics," Kenneth Rogoff
- "Protection and Real Wages: The Stolper–Samuelson Theorem," Rachel McCulloch

Avinash Dixit provides an appraisal of Samuelson's work on trade theory. He explains the scientific importance of starting with the Ricardian 2×2 model, and the current literature on comparative advantage. Samuelson rests this model on the shoulders of giants, whether we want to explain gains from trade by swapping bananas for steel, or from the modern theoretical points of view.

Samuelson has picked up this model without any rigorous proof and has given it many "operational" assumptions so that one can test—or falsify—its predictive or explanatory powers. He provides this intuitive proof: "Anybody can see that the tropics are capable of producing bananas while the temperate regions cannot produce bananas but can produce steel. Thus there would be profitable interchange between the cold, northern region, and the tropical region. That's the theory of comparative advantage" (Samuelson, 1986, p. 52).

In modern times this cannot be improved too much from the intuitive side. Dixit and Norman (1980, p. 5), for instance, argue that "the concepts involved are imprecise," and therefore "it is far from trivial to establish them rigorously." More advanced proof led to the welfare gain concept that Samuelson advanced from the classics, and anchored squarely on the First and Second Welfare theorems. With these theories, we can now model trade to assess who gains, for instance, in the formation of a Free Trade Agreement (FTA). This kind of research has only just begun.

Rogoff's chapter, which addresses the contemporary policy debate, appraises Samuelson's trade contributions such as the Stolper–Samuelson and factor–prices equalization theorems as "vital in today's globalization debate." The idea that gains from trade can be modeled through side-payment, that Samuelson added intuitive understanding and easy testing of the Stolper–Samuelson theory, and that his simple "iceberg-cost" metaphor helped our understanding of transit cost and friction in trade, are very active in the modern scientific and development views of modern trade theory. Rogoff's chapter spans a wide range of thought, from the "iceberg-cost" concepts to a Ricardian "continuum of goods" trade model. In between are financial analysis models such as the Harrod–Balassa–Samuelson theorem that the exchange rate increases faster for growing countries, which has led to the development of the Heston–Summer database for world comparison

of income and prices. In between is his contribution on the "Transfer Problem," which at that time proved Keynes correct in his prediction of the cost of Germany's postwar reparations, and is still used today to assess the effects of a trade deficit on trade.

McCulloch appraises the Stolper and Samuelson (1941) theorem under varying conditions and assumptions. The model predicts that a country will export commodities produced by its abundant factors, and will import commodities produced by its scarce factors. The consequence is that trade will lower the real wages of the scarce factors.

In the current milieu of free trade, factors can move from import competing to export industries. In that movement, the factors including labor may lose real income, independent of taste and spending patterns. Using Jones' reformulation of the theorem, McCulloch demonstrates that when the relative prices for labor-intensive goods fall, real wages will decline in that sector, and the returns to other factors will increase, resulting in a redistribution effect.

The model had been robust in its predictions. When the number of goods and factors are increased, at least one factor is likely to gain or lose. The model is relevant for modern policies such as the Trade Adjustment Assistance (TAA) and unemployment insurance programs that assist distressed industries, where compensation can make a difference for individuals or firms that are hurt from free trade. The model is also relevant in the political arena where owners of factors vote or lobby. It has shown strength in explaining patterns of protection across countries. We see that the model had a long life in explaining distributive shares of factors, an area that will undoubtedly occupy social scientists in the twenty-first century.

Finance and Portfolio Theory

There are three chapters in this section:

- "Samuelson and the Factor Bias of Technology Change," Joseph Stiglitz
- "Samuelson and Investment for the Long Run," Harry M. Markowitz
- "Paul Samuelson and Financial Economics," Robert C. Merton

Stiglitz's chapter appraises a simple model Samuelson enunciated over 40 years ago on the liberalization of the capital markets. The model finds a

home in the globalization of modern capital markets, indicating Samuelson's anticipation of free trade ahead of its time. The story is that unlike the situation of OLG where capital markets without liberalization but with technology shock will transfer over to another generation, with liberalization the shock may be dissipated in the current generation.

Stiglitz appraises the capital liberating model first from the traditional equilibrium points of view, and then from his new paradigm of disequilibria or market imperfection points of view. In the equilibrium version, some ambiguity exists as to how technological progress would augment capital or labor. Kaldor's stylized facts approach had assumed away the problem. The standard Harrod–Domar model did not include the effect of technological change, and when it was added, the disequilibria between exogeneous labor, and adjusted warranted growth rate becomes clear. Solow's modification did improve the analysis by making capital and effective labor grow at the same rate, but at the price of diminishing the concept of a job.

Stiglitz's appraisal of the capital liberating model now points out that wage adjustments with given capital and technology can support only maximum employment, and not necessarily lead to full employment. He provides two versions. A fixed coefficient version that extends Samuelson's paradigm to include wage effect through technology and capital accumulation on employment, and a version based on agency theory or efficiency wage theory where the wage rate must be adequate to induce labor to work. By integrating efficiency wage theory into the capital liberalizing model, Stiglitz has shown that Samuelson's model will be significant for future research in explaining dynamic economic problems.

Markowitz appraises a debate with Samuelson regarding which criteria the long-run investor should maximize in their portfolio. Markowitz provides the example of receiving either 6 percent per year with certainty or a lottery with an equal chance of 200 percent gain or 100 percent loss each year. The expected value returns criteria yields a return of 0.5. The expected log of 1 plus the returns is negative infinity ($-\infty$). Therefore, the investor would choose the certain prospect.

Markowitz considers whether the long run investor should follow the arithmetic mean or the log arithmetic (geometric mean) criteria in maximizing its portfolio. Markowitz makes the case for the log model. Samuelson argues that "It is a mistake to think that, just because a w^{**} decision ends up with almost-certain probability to be better than a w^{*} decision, this implies that w^{**} must yield a better expected value of utility"

(Samuelson, 1986, p. 246). He complemented this argument with an intransitive rule of odds: "You may well pick A(N) from A(N) and B(N), and pick B(N) from B(N) and C(N), and yet still pick C(N) from A(N) and C(N)" (Samuelson, 1986, p. 554). Nevertheless, the Markowitz chapter continues to present the case for the log model.

Merton's appraisal identifies essential and substantive gems from Samuelson's original contributions to financial economics. He selects financial facts that form a synthesis, implying that some conflicts have been resolved about them. Merton found that much of Samuelson's contributions, which he had appraised 25 years ago, are even more significant today.

Merton's appraisal places models of time and uncertainty in household allocation of resources at the center of the Samuelson contribution. He assesses Samuelson's contributions to the areas of efficient market theory and risk analysis, portfolio selection, and option and warrant pricing. Samuelson found that efficient markets do not allocate resources the way casinos do. Rather, asset prices vary randomly around an optimal path that can be discerned mathematically. The theory links space (spot price) with time (current future prices) in order to forge a solution (current futures prices = future spot prices), where spot prices are determined by optimal control theory. Can a George Soros, for example, provide superior performance? The answer depends on whether he can explain variation around expected returns. Operationally, that depends on the amount of information he has. The ranking of information produces strong, semi-strong, and weak versions of the efficiency hypothesis. Technically speaking, the matter of superior performance requires explaining the difference between a random variable and its conditional expectation, a difference with martingale properties. Researchers today are testing for these properties, and will continue to do so in the future.

Prior to Samuelson's works, options pricing centered mainly on European options, defined as options exercised on their expiration date. Samuelson introduced the American option that can be exercised before that date, and he considered longer-term horizons as well. For long-held or perpetual options, he discovered that the option would sell for more than the value of the stock. To correct that anomaly, he introduced log values that eliminated negative terms in stock pricing calculations. In short, Samuelson's work in warrants and option pricing provide a bridge between early and later option pricing models, thanks to his insights on hedging and mathematical analysis that were incorporated into subsequent theories.

Merton further appraises Samuelson's financial contribution for some other long-run issues. Samuelson introduced very specialized utility functions to address age-dependent and risk issues relating to, for instance, the notion that stocks are not risky in the long run. The argument for that proposition was based on some empirical fact on returns over 15- or 20-years horizon, and the argument that people become more conservative with age. Samuelson reached the conclusion that stocks are risky in short, intermediate, and long runs, and therefore, he leans toward the rejection of that hypothesis. Not heeding Samuelson's advice, retirement investment during the 2000–2002 period has experienced much loss.

Samuelson's Relevance

There are three chapters in this section:

- "Multipliers and the LeChatelier Principle," Paul Milgrom
- "The Surprising Ubiquity of the Samuelson Configuration: Paul Samuelson and the Natural Sciences," James B. Cooper and Thomas Russell
- "Paul Samuelson's Mach," Rod Cross

Milgrom appraises Samuelson's LeChatelier principle of how the market responds to a change in parameters of demand and supply curves. He uses examples in demand theory, economic policy, and empirical research to illustrate the principle. He also stresses the flexibility of the principle to adapt to changing assumptions. We notice changes from the optimizing agents to equilibrium systems whose primary use is to "provide a foundation for understanding multipliers."

Milgrom evaluates how the principle performed when it was confronted with local optimization problems as in production function settings, and in positive feedback systems as in gaming situations. The principle is found progressive in that it is able to capitalize on symmetric relations among substitutes and complements. In that regard, the principle has extended research into multiplier analysis, a research area that would continue into the twenty-first century.

Cooper and Russell appraise one aspect of how Samuelson adopted the methods of physics for economics. Samuelson is considered a leader in adopting the concepts of optimization with constraints to economics.

Cooper and Russell grasped the source of this principle in the "little used" and "amazingly obscure" writings of the physicist James Clerk Maxwell.

Cooper and Russell's appraisal delves deeply into the areas of physics and mathematics. "Classical thermodynamics is the subject that deals with such interactions of mechanics and 'temperatures' . . . my formulation . . . is idiosyncratic in that the formal relations that are important in analytical economics have motivated my choice of physical axioms and the order of their introduction." In thermodynamics, Samuelson "utilize[s] the area of a closed curve in the (pressure, volume) plane." In economic analysis, we use the terms price and quantity to replace pressure and volume, respectively. In such a diagram, equivalent areas between level curves can be interpreted as solutions to the maximization of profit subject to input constraints problems. From the area of classical mechanics, Samuelson borrowed "the law of conservation of (mechanical) energy for a (frictionless) system." A frictionless pendulum has the greatest kinetic energy (squared velocity) at the bottom of its swing, when its potential energy is minimal. The sum of a conservative system's kinetic and potential energies is constant along any motion, conserving the initial value of that sum (Samuelson, 1986, pp. 231–232). This conservation model has been a workhorse in modern economics in optimization problem, and shows no sign of weakening in the twenty-first century.

Cross appraises Samuelson's methodology through the work of the physicist, psychologist, and philosopher Ernst Mach, categorizing such thoughts with a big "M". The range of thought of Mach is from sensory observation to phenomenology, including other disciplines such as psychology to the extent that such disciplinary thoughts are in harmony with the stability of the concept.

Cross appraises Samuelson from the Heraclitus dictum to the effect that one cannot step into the same river twice because of changes that have taken place. The river has essential properties that do not change, and substantive properties that do change from an Aristotelian viewpoint. For Samuelson, "Science, even inexact science is public knowledge, reproducible for analysis by everyone" (Samuelson, 1986, p. 564). Cross locates that reproducibility in the "action and reactions" of the elemental qualities of a river, or relationship among economic variables. When the qualities such as *a, b, c* satisfy a functional form such as $F(a,b,c \ldots) = 0$, then they come together at that point, forming a state between appearance and disappearance. States have recall properties that allow association of past observations with current observations, enabling scientific discovery

to take place. Cross appraises how Samuelson meandered among the thoughts of his teachers, colleagues, theoretical and empirical concepts to place his final allegiance on facts as the pivot on which to gauge scientific economic theories.

Cross ventures that Samuelson's methodology arises from the consideration of many views of Science. Faced with a blurred distinction between facts and theory that was highlighted by W. V. Quine, and the argument that economics deals with the world of social phenomena, Samuelson maintains a firm foot in reality, letting the facts tell their story. Samuelson also acknowledges how new thoughts such as Thomas Kuhn's paradigm, regarding the notions of cumulative knowledge, and incommensurability brings to the theory of science. He accepted that facts are numerous and therefore must be carefully sampled for their economy. But his goal remains that we should be able to tell the "how" and the "why" of things and phenomena of the economic world.

Samuelson's relevance starts with his curiosity for finding simple facts. He wrote that "The simplest things are often the most complicated to understand fully." He suggests an approach that treats simple things with: "(1) A literary discussion; (2) A mathematical treatment, and finally; (3) A history of the subject" (Samuelson, 1978, p. 3). Even in the area of mathematics, which has the reputation of being precise, he reminds us that "When mathematicians, like Debreu, speak of a competitive equilibrium, they do not insist that it is to be the only one but merely that it be self-warranting in the sense of satisfying all the conditions of the problem" (Samuelson, 1978, p. 143). In the same vein, he writes that "If the solution is simple, the assumptions must be heroic . . . Assumptions would not be heroic if they could be easily taken for granted as being exactly applicable" (Samuelson, 1978, p. 150). In one sense, we can think of the term "generalist" as a polymath—that is, one who has an umbrella of concepts under which to predicate particular concepts. From this point of view, Samuelson reaches for infinity as the limit. His search for the truth is far reaching. It is worth repeating his famous quote: "I would take aid from the Devil if that would help crack the puzzle of economic reality" (Samuelson, 1986, p. 873). Again, he is willing to look at the recesses of the subconscious to find the truth if need be: "We are eternally grateful to Henri Poincare for his detailed exposition of the role that the subconscious plays in the discovery of mathematical theories: how one wrestles consciously and unsuccessfully with a theorem, then puts it aside, as if out of mind, but apparently not really out of mind; for suddenly . . . the successful solution arrives" (Samuelson, 1978, p. 846). From Samuelson's generalist point of

view, the truth can be approached from anywhere. "A mathematical theoretical Walras–Debreu system would find a full-employment equilibrium path even if it started out from initial conditions like those of 1933" (Samuelson, 1978, p. 915).[5] To appraise Samuelson as a generalist is to put him in the heart of research for the twenty-first century. In the twentieth century, the physical scientists could tell what to do and where to go if some object was hurled at you, needing only Newtonian laws. As we enter the new millennium, we are bombarded with more uncertainty in science. Just think of such phenomena as global warming, parallel universes, antimatter, and spin theory.

Samuelson's "generalist" approach seems to be right at home with the general view of the physical sciences.[6] In his *Foundations of Economic Analysis* (1947), Samuelson looked for similarities among the various areas of economics. He then proceeded as a "generalist" to bind them together. This is in the time-honored canonist approach to scientific discovery extolled by Bacon and Mill, of which we list the First Cannon according to Mill: "*If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon*" [Italics original] (Mill, 1970, p. 255). To do that for economics, Samuelson prescribed the tools of a precise language, which he found in mathematics. He also prescribed and invigorated concepts such as maximization, minimization, equilibrium, efficiency, stability, and multiplier. With these generalized concepts came new ideas in the areas of cost, production, consumer demand, revealed preference, and trade in their static and dynamic points of view.

Samuelson is explicit and conscious about his position as a generalist, for he notes "I can claim that in talking about modern economics I am talking about me. My finger has been in every pie. I once claimed to be the last generalist in economics" (Samuelson, 1986, p. 800). Without proper appreciation of Samuelson's "generalist" perspectives, many analysts who have attempted to appraise Samuelson's work, not necessarily for the twenty-first century, have not been able to contain their analysis to the scientific domain, but rather were led into directions dictated by their own partiality. For instance, in reviewing the contribution to utility theory, Hayek was led to the view that "refinements suggested by P. A. Samuelson are hardly in the Austrian tradition" (Hayek, 1992, p. 54). But even Hayek too sought the general, not reasoned, approach, which he found in the theory of evolution. How democratic socialism can evolve into totalitarianism is the thesis of Hayek's *Road to Serfdom* (1945). Even though

the Austrians are apt to argue that Samuelson has adopted the wrong method, "establishing physics as the science for economics to imitate," (Hayek, 1992, p. 5) their sympathizer, Robbins (1970, pp. 40–41), had this to say of Samuelson's methods: "It is difficult to argue that . . . the comprehensive treatise of Dorfman, Samuelson and Solow have not deepened our insights in many directions . . . in the raison d'etre of the price mechanism as something inherent in any maximization process within the restraints of different degrees of scarcity."

Conclusion

In this introduction, we reviewed the significance of Samuelson in the twenty-first century using the scientific criteria of "unit" and "units of appraisal." Our appraisal suggests continued success for Samuelson's specific and general studies for the twenty-first century. His work on trade is finding applications in the modern global economy. While his writings on the neoclassical synthesis were put to severe tests (and some parts shaken) during the 1970s and 1980s, they have an increasing role to play in the modern global economy. A mixed economy and public versus private goods with their associated externalities are very much twenty-first century concepts. Samuelson's theoretical contributions, when given operational meaning, are increasingly being confronted with facts and reality, and are performing well. Overall, the appraisals place a high value on Samuelson's vast output from a scientific point of view, which supports their endurance in the years ahead.

Notes

1. To appraise Samuelson, consider his preference for mixing fiscal and monetary policies. Samuelson wrote: "Now, I will very briefly summarize my view on the subject. The late C.O. Hardy said, 'Fiscal policy really has not independent importance. It is just a complicated way of getting the banking system to create some extra money. It is like burning the house in order to roast a pig' " (Samuelson, 1972, p. 552). This story falls in with Dewey's *Propositions of Appraisal*, where an "examination of these appraisals discloses that they have to do with things as they sustain to each other the relation of *means to ends or consequences*" (Dewey, 1939, p. 23). One interpretation of this "means to ends" model is that one should skip fiscal policy and get to where the action is—monetary policy. This will be an instance of "the end justifies the means"

(Dewey, 1939, p. 41). Samuelson thinks, "this view is profoundly wrong . . . that the mixture of fiscal policy and monetary policy we actually use was absolutely crucial in this and other regards" (Samuelson, 1972, p. 552). Such policy mix has already made its way into the current economic thinking that argues for monetary targets when the IS curve shifts about, or fixed interest rate policy if the LM curve shifts about (Dornbusch, 2004, p. 426).

2. Samuelson appears to substitute the term "audit" for "appraisal" at times: "What does an audit show for these opposite-line claims? Does it confirm any hope to explain the trends toward deterioration of the double factorial terms of trade of the Third World vis-à-vis the affluent nations by means of the concept of unequal exchange?" (Samuelson, 1986, p. 477). Also, he used the synonym "analyze" from time to time: "Here in this brief investigation I hope to analyze what the effects are on the welfare of different regions of a great burgeoning of productivity in the Pacific Basin" (Samuelson, 1986, p. 484), or: "I shall be analyzing the merits and demerits of protection" (Samuelson, 1986, p. 493), and the synonym "evaluate" as well: "To evaluate the question of how different the classical paradigm was from today's mainstream economics, it is worth sketching briefly the consequences of replacing *f* (Min [L,K]) by smooth constant-returns-to-scale technology" (Samuelson, 1986, pp. 606–607).
3. Note that Popper (1962) says that to appraise is not to accumulate knowledge but to replace one paradigm with another.
4. The longevity of Samuelson's model is assured in the twenty-first century and beyond. For as long as marriage and divorce will be around, the need for utility function inside and outside of family relationships will be needed to assess benefits and losses. The paper also appraises the pitfall of Samuelson's model, indicating the pros and cons of various empirical applications that are currently being carried out.
5. This should not be taken to imply that economics is all about common sense. Samuelson was clear about this matter when he wrote that "While it is true that few with advanced training in economics can be trusted to use common-sense economics, fewer still, and maybe no one, without advanced training in economics, can be trusted to use common-sense economics . . . experience does show that the best economic policy-makers have spent years studying economics and doing scientific research . . . Common sense, and folklore generally, lack empirical content . . . I would liken common sense to the hands of a watch, hands so short that they lie in every direction; lying in every direction, the hands cannot point in any direction and such a watch can tell us the correct time only after we have already learned it elsewhere" (Samuelson, 1962, pp. 16–17).
6. We see this as compatible with Adam Smith's intention when he set out to do for the social sciences what Newton did for the physical sciences, and predict that the foundation that Samuelson laid for the twenty-first century is a progressive one.

References

Allais, Maurice, *Economie et intérêt*, (Paris: Imprimerie Nationale, 1947).

Arrow, Kenneth, "Samuelson Collected," *Journal of Political Economy*, Volume 75, No. 5, (October 1967), 730–737.

Bateson, Gregory, "Experiments in Thinking about Observed Ethnological Material," *Philosophy of Science*, Vol. 8, No. 1, (January 1941), 53–68.

Bateson, Gregory, and Margaret Mead, "Principles of Moral Building," *Journal of Educational Sociology*, Vol. 15, No. 4, (December 1941), 205–220.

Blaug, Mark, *The Methodology of Economics*, (Cambridge University Press, 1983).

De Marchi, N., "Anomaly and the Development of Economics" in *Method and Appraisal in Economics*, Spiro Latsis, editor, (Cambridge University Press, 1976).

Dewey, John, "Theory of Valuation," *International Encyclopedia of United Science*, Volume II, Number 4, (The University of Chicago Press, 1939).

Dixit, Avinash K. and Victor D. Norman, "Theory of International Trade. Cambridge", (Cambridge University Press, 1980).

Dornbusch, Rudiger, Stanley Fischer, and Richard Startz, *Macroeconomics*, Ninth Edition, (New York: McGraw-Hill Irwin, 2004).

Feiwel, George F., *Samuelson and Neoclassical Economics*, (Boston: Kluwer-Nijhoff, 1982).

Hahn, Frank and Robert Solow, *A Critical Essay on Modern Macroeconomic Theory*, (Cambridge, MA: The MIT Press, 1995).

Hayek, F. A., *Collected Works*, Volume 4, (The University of Chicago Press, 1992).

Heilbroner, Robert and William Milberg, *The Crisis of Vision in Modern Economic Thought*, (Cambridge University Press, 1995).

Hicks, John, *Collected Essays on Economic Theory: Classics and Moderns*, Volume III, (Harvard University Press, 1983).

Hildenbrand, Werner, *Core and Equilibria of A Large Economy*, (Princeton University Press, 1974).

Hutchison, T. W., "On the History and Philosophy of Science and Economics," in *Method and Appraisal in Economics*, Spiro Latsis, editor, (Cambridge University Press, 1980).

Lakatos, Imre, and Alan Musgrave, eds., *Criticism and the Growth of Knowledge*. (Cambridge University Press, 1977).

Lakatos, Imre, *The Methodology of Scientific Research Programmes: Philosophical Papers* Volume I, Edited by John Worrall and Gregory Currie, (Cambridge University Press, 1980).

Latsis, Spiro, "Situational Determinism in Economics," *The British Journal for the Philosophy of Science*. 1972, Volume 23, 207–245.

Latsis, Spiro, *Method and Appraisal in Economics* (Cambridge University Press, 1980).

Machlup, Fritz, *Methodology of Economics and Other Social Sciences,* (New York: Academic Press, 1978).

Malinvaud, Edmond, "The Overlapping Generation Model in 1947," *Journal of Economic Literature*, Vol. 25, No. 1, (March 1987), 103–105.

Mill, John Stuart, *A System of Logic*, (London: Longman, New Impression 1970).
Miller, Authur I., *Imagery in Scientific Thought: Creating 20th Century Physics*, (Boston: Birkhauser, 1984).
Poincare, Henri, *Science and Method*, (New York: Dover, n.d.) translated by F. Maitland from *Poincare's Science et Methode*, (Paris: Flammarion, 1908).
Polanyi, Michael, *Personal Knowledge*, (University of Chicago Press, 1958).
Popper, Karl R., *The Logic of Scientific Discovery*, (New York: Harper Torchbooks, Harper & Row, Publisher, 1968).
Popper, K. R., *Conjectures and Refutations*, (New York: Basic Books, 1963).
Quine, W. V., *Pursuit of Truth*, (Cambridge, MA: Harvard University Press, 1990).
Ravetz, Jerome R., *Scientific Knowledge and its Social Problems*, (Oxford University Press, 1979).
Robbins, Lionel, *The Evolution of Modern Economics Theory*, (London: MacMillan, 1970).
Samuelson, Paul A., *Foundations of Economic Analysis*, (Harvard Economic Studies, Volume 80, 1947, New York: Atheneum 1974).
Samuelson, Paul A., *Problem of the American Economy*, *The Stamp Memorial Lecture 1961*, (London: The Athlone Press, 1962).
Samuelson, Paul A., "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *The Journal of Political Economy*, Volume LXVI, No. 6, (December 1958), 467–482.
Samuelson, Paul A., *Collected Scientific Papers of Paul A. Samuelson*, edited by Joseph E. Stiglitz, (Cambridge, MA: The MIT Press, Volume 1 and 2, 1966).
Samuelson, Paul A., "Collected Scientific Papers of Paul A. Samuelson," edited by Robert C. Merton, (Cambridge, MA: The MIT Press, Volume 3, 1972).
Samuelson, Paul A., *The Collected Scientific Papers of Paul A. Samuelson*, edited by H. Nagatani and K. Crowley, Volume 4, (Cambridge, MA: MIT Press, 1978).
Samuelson, Paul A., *Collected Scientific Papers of Paul A. Samuelson*, edited by Kate Crowley, (Cambridge, MA: The MIT Press, Volume 5, 1986).
Samuelson, Paul A., "My Life Philosophy: Policy Credos and Working Ways," in Michael Szenberg, ed., *Eminent Economists, Their Life Philosophies*, (Cambridge University Press, 1993).
Samuelson, Paul A., "Foreword," in Michael Szenberg, ed., *Passion and Craft, Economists at Work*, (Ann Arbor, MI: University of Michigan Press, 1999).
Sraffa, P., *Production of Commodities by Means of Commodities*, (Cambridge University Press, 1960).
Szenberg, Michael, Aron A. Gottesman and Lall Ramrattan, *Paul Samuelson on Being an Economist* with a Foreword by Joseph E. Stiglitz, (New York: Jorge Pinto Books, Inc., 2005).
von Mises, Ludwig, *Epistemological Problems of Economics*, (New York: D. Van Nostrand Company, Inc., 1960).
Walker, Donald W., *William Jaffe's Essays on Walras*, (Cambridge University Press, 1983).